The University of British Columbia

ELECTRONICS II

ELEC 301

MINI PROJECT 4

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Contents

1 Objectives

To become familiar with and understand the basic operation of active filters and oscillators and to understand some of the characteristics of feedback amplifiers

2 Part I: An Active Filter

2.a Locking Down Values: The Butterworth Filter

Below we have an active filter circuit which we will turn into a 2nd order Butterworth filter. Using a UA741 op-amp, a 15V power supply, and knowing $A_m = 1+\frac{R_2}{R_1}$, $R = R_1+R_2 = 10k\Omega$, we can wire up this circuit.

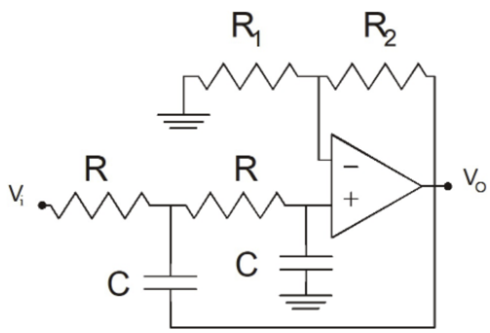

Figure 2.1: A 2nd order low pass filter

With the proper selection of the DC gain, the circuit shown in Figure 2.1 is a 2nd order low-pass filter. The transfer function is:

$$H(s)=A_M\frac{1/(RC)^2}{s^2+s\frac{3-A_M}{RC}+\frac{1}{(RC)^2}}$$

Figure 2.2: Cascode Amplifier Specifications

Recalling second order Butterworth filter polynomials are in the form $s^2+2\zeta s+1$, where $2\zeta = 1.414$ is the damping factor, we can use the relation $A_m = 1+\frac{R_2}{R_1} = 3-2\zeta$ to solve for the resistances:

$$A_m = 1+\frac{R_2}{R_1} = 3-1.414 = 1.586 \quad \rightarrow \quad 0.586 = \frac{R_2}{R_1}\ ,\ R_1+R_2 = 10k\Omega$$

$$\text{solve} \rightarrow \quad R_1 = \boxed{6.3\text{k}} \quad R_2 = \boxed{3.7\text{k}}$$

Given a 3dB frequency of 10kHz, capacitance can be calculated in a Butterworth filter using the cutoff frequency:

$$\omega_C = \frac{1}{RC} \quad \rightarrow \quad C = \frac{1}{\omega_C * R} = \frac{1}{2*\pi*10kHz*10k\Omega} = \boxed{1.6\text{nF}}$$

Applying the resistance and capacitor values to Figure 2.1 and taking the Bode plot:

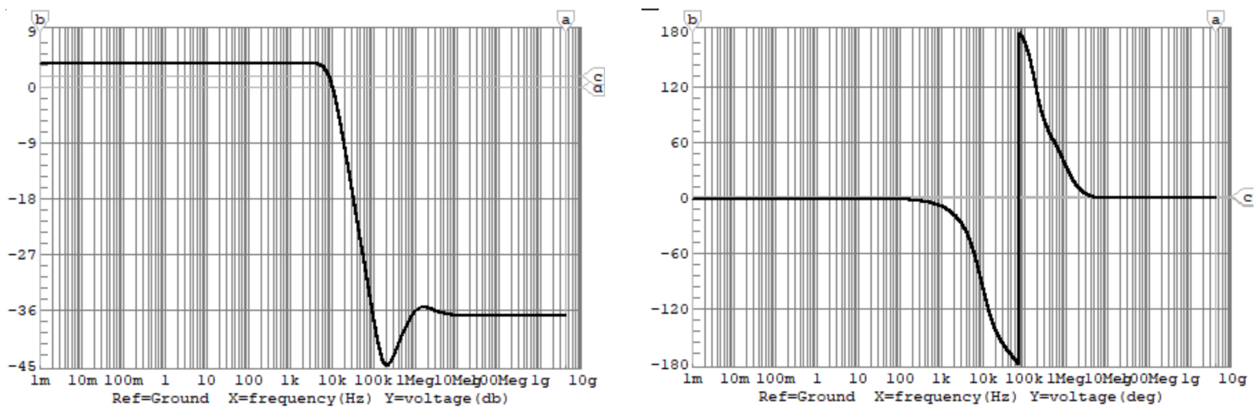

Figure 2.3: Bode Magnitude plot (left) and Bode Phase Plot (right) of Figure 2.1

2.b Finding Instability via Oscillation

To find where the system starts oscillating we will first connect the input to ground. We will then increase the gain A_M of the system by slowly increasing R_2 and decreasing R_1 while keeping the sum of the two $10k\Omega$. We eventually find the system oscillates after around 16ms at $R_2 = \boxed{6.7\text{k}}$, $R_1 = \boxed{3.3\text{k}}$ in testing.

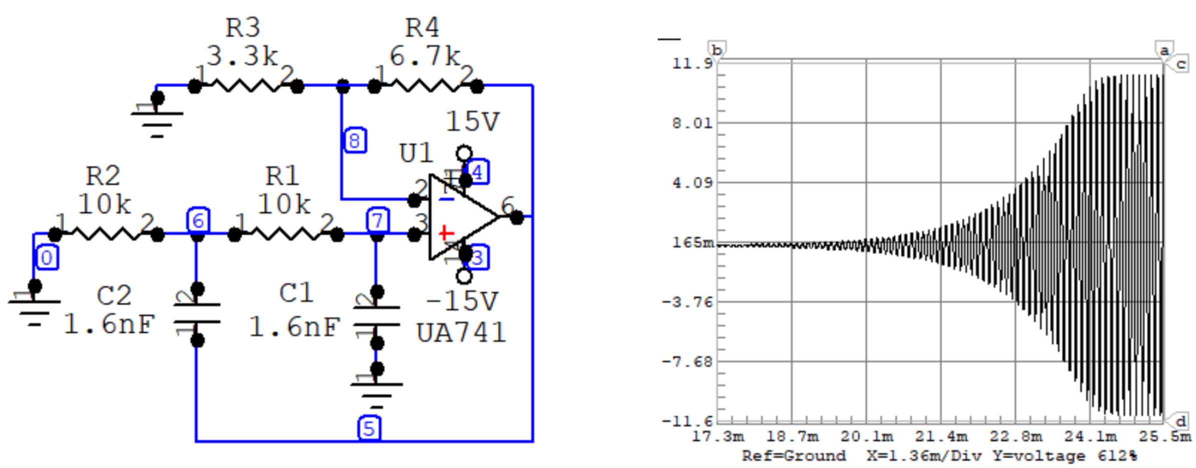

Figure 2.4: 2nd Order Active Filter (left) and it's Transient Plot (right)

Zooming in on our oscillating circuit's transient we find that it has a period of $T = 111u$ which is a frequency $f = \boxed{9\text{kHz}}$

2.c Discussion

Do these resistance values from 2.b make sense? To test we will calculate the point at which the system should become unstable. To do this we will set $\zeta = 0$, making the system undamped, which will signal the start of system instability. We can see this in the root locus plots below. The root locus of our 2nd order Butterworth filter is on the left and root locus for our underdamped oscillating filter is on the right.

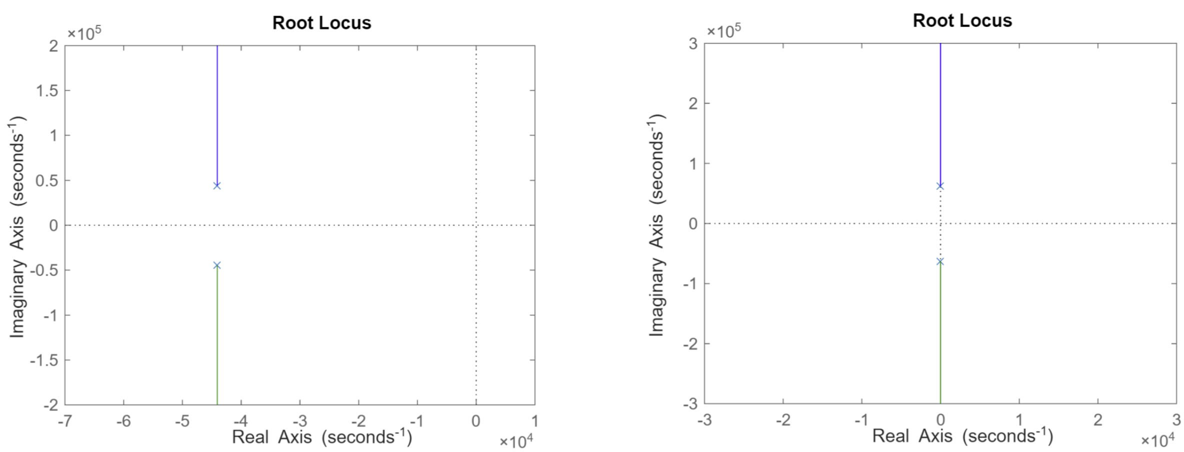

Figure 2.5: Root locus when $2\zeta = 1.41$ (left) and when $2\zeta = 0$ (right) for our active filter

Using the resistance ratio and knowing our max resistance we can find the values which make the system oscillate.

$$2\zeta = 2 - \frac{R_2}{R_1} = 0 \quad \rightarrow \quad 2 = \frac{R_2}{R_1} = \frac{6.666k\Omega}{3.333k\Omega} \quad \rightarrow \quad R_1 = \boxed{3.3\text{k}} \quad R_2 = \boxed{6.7\text{k}}$$

Our resistances make sense! This also makes intuitive sense if look at the transfer function from Figure 2.2, a resistance ratio of 2 means a gain of 3 which would cancel out the middle term in the denominator. These values line up with out tested values so our initial analysis was correct.

3 A Phase Shift Oscillator

The phase shift oscillator is an active circuit combination that uses regenerative feedback to produce a sinusoidal output without any input. The initial voltage is provided by input noise from the voltage applied to the op-amp. The "phase shift" in phase shift oscillator is referring to the two 180° degree phase shifts; one from the 3 highpass filters and one from the op-amp output. The capacitors effectively work to cancel out the 180° degree phase that comes from the op-amp, which results constructive interference of the waves. This is compounded and goes on exponentially until maximum voltage is output from the input rails of the op-amp. Circuit shown in Figure 3.1.

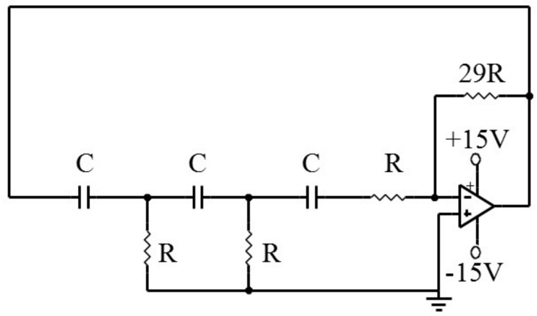

Figure 3.1 Phase Shift Oscillator

The strange 29R resistor value in the circuit serves as a gain mechanism. It increases the gain by 29X from op-amp input. This is important as after the feedback signal passes through the capacitor/resistor portion it comes back attenuated to $\frac{1}{29}$ the feedback input. The 29R is a minimum gain requirement to begin the exponential signal growth but I will use 30R to increase the time it takes for the transient to stabilize.

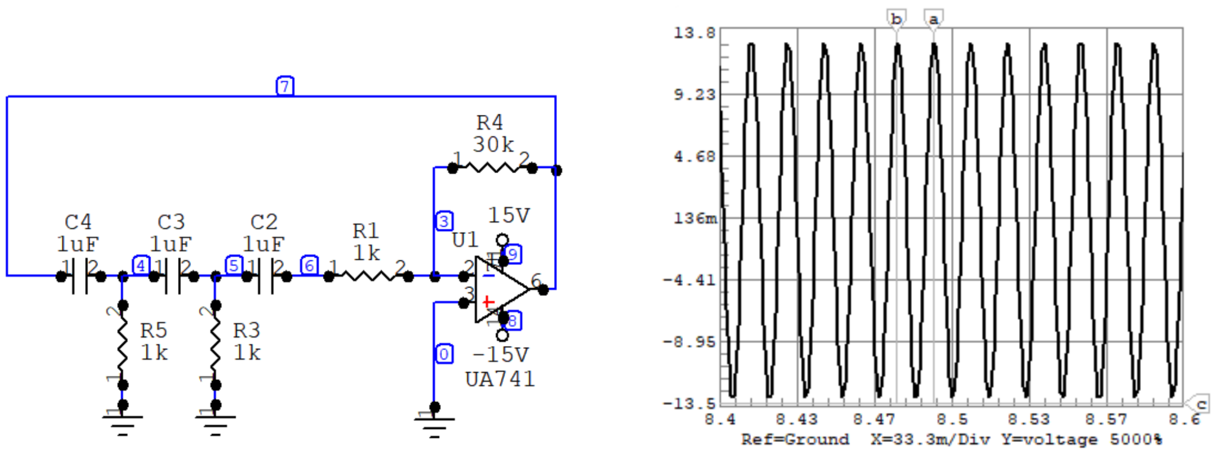

Figure 3.2: Phase Shift Oscillator with Values and Transient Response

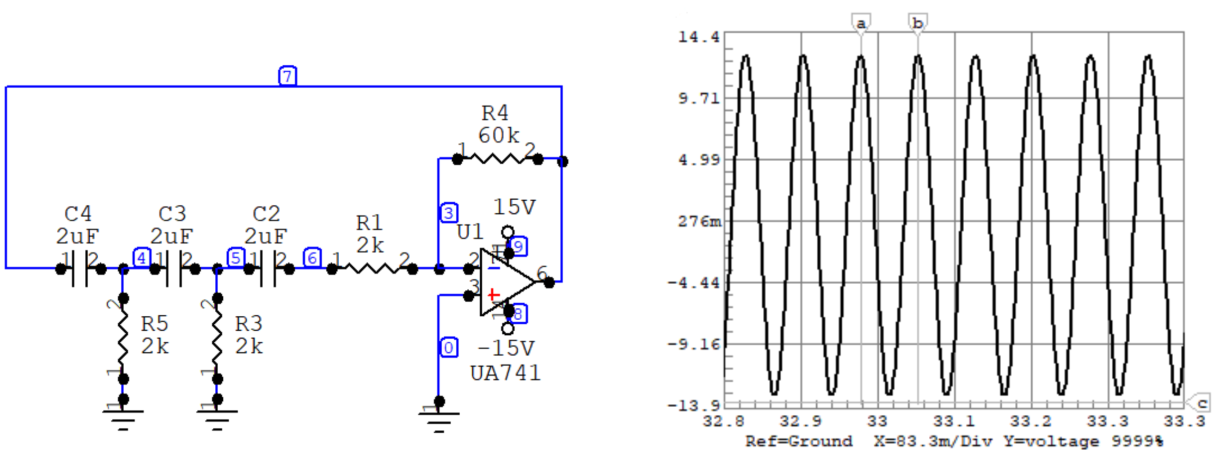

Figure 3.3: Phase Shift Oscillator with 2x Figure 3.2 Values and Transient Response

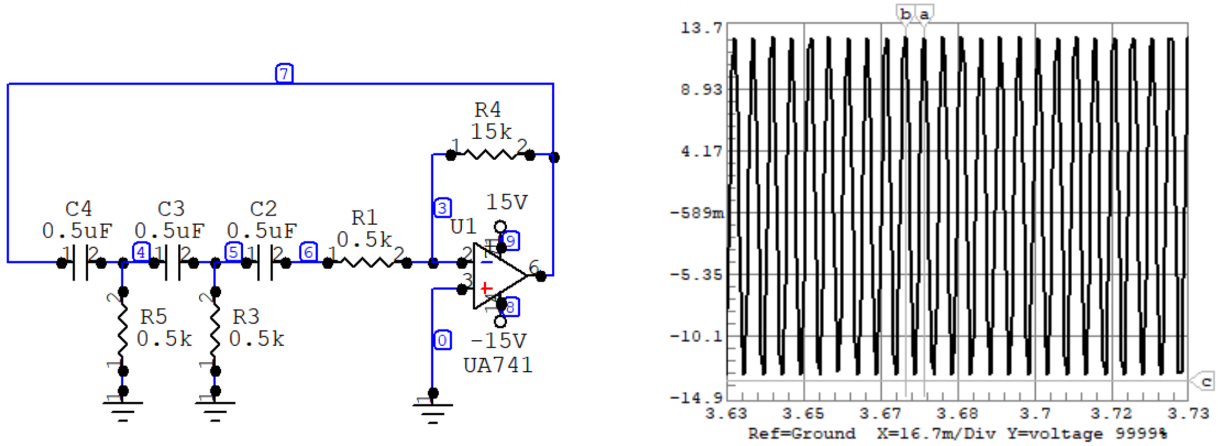

Figure 3.4: Phase Shift Oscillator with 0.5x Figure 3.2 Values and Transient Response

From each of the transient responses we can identify the period and find the frequency:

Circuit	Measured Frequency
Default (Figure 3.2)	62.50 Hz
2x Default (Figure 3.3)	16.02 Hz
0.5x Default (Figure 3.4)	247.43 Hz

Table 3.1: Simulated Frequency of Phase Shift Oscillators

This is one way of finding a frequency that we want for our circuit: guess and test. However, there is a more effective way, we can use the formula $f = \frac{1}{2\pi\sqrt{6}RC}$:

Circuit	Calculated Frequency
Default (Figure 3.2)	65.97 Hz
2x Default (Figure 3.3)	16.24 Hz
0.5x Default (Figure 3.4)	259.90 Hz

Table 3.2: Calculated Frequency of Phase Shift Oscillators

The discrepancy between the frequency values that were simulated from CircuitMaker and the values that were calculated using the formula are negligible and can likely be associated with the CircuitMaker software graphing UI and human error of estimation.

4 A Feedback Circuit

Next we will explore the operation of a feedback circuit. Two important aspects of the feedback network are how the signal is mixed (subtracted) at the input and how it is sensed at the output. Only four basic mixing and sensing schemes are possible since both mixing and sensing are done either in series or in shunt.

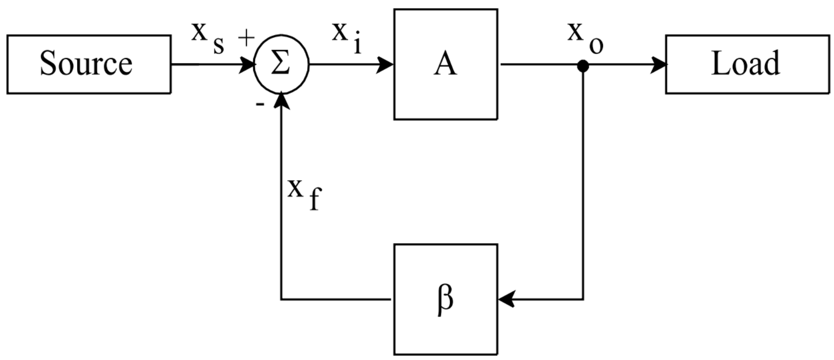

Figure 4.1: A Basic Feedback Network

Wiring up a circuit based on the Figure 4.1 diagram, we can create something like Figure 4.2. We will use use 2N3904 transistors, and vary R_{B2} to find the ideal resistance for the largest open loop gain at 1kHz. First leaving R_F open.

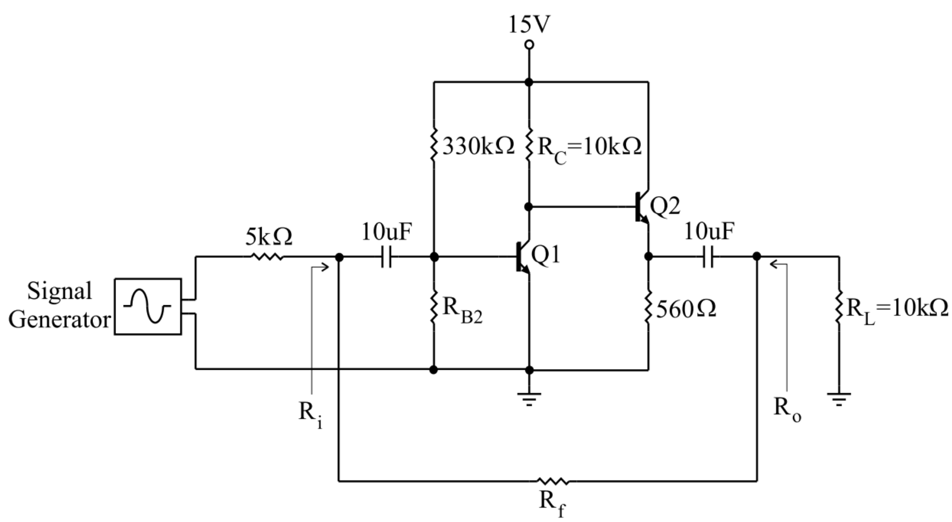

Figure 4.2: The Feedback Circuit

Resistance Value R_{R2}	10kΩ	20kΩ	30kΩ	40kΩ	50kΩ
Open Loop Gain @1kHz	-21.73 dB	42.13 dB	-21.73 dB	-101.31 dB	-106.22 dB

Table 4.1: Calculated Frequency of Phase Shift Oscillators

Finding our high gain resistor we can wire up our new circuit and find DC bias values for each transistor.

Methods	V_C	V_B	V_E	I_C	I_B	I_E
Q1	1.9V	0.65V	0V	1.26mA	10.77uA	1.31mA
Q2	15V	1.9V	1.24V	2.19mA	15.39uA	2.21mA

Table 4.2: DC Operating Points

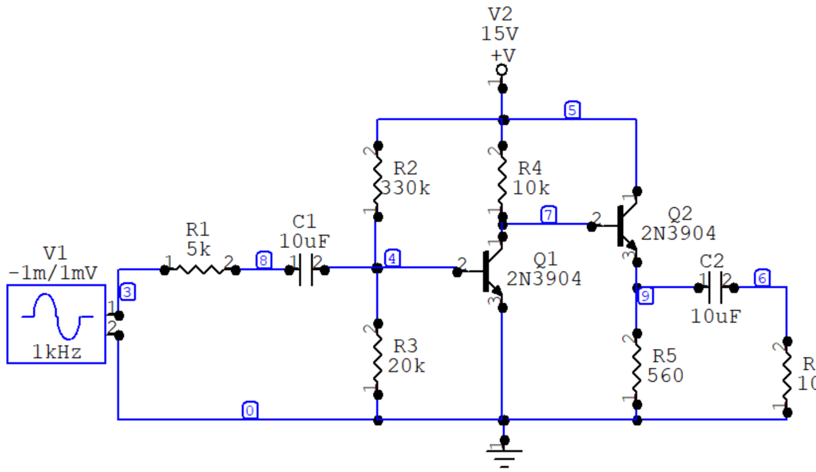

Figure 4.3: Figure 4.3 with R_{B2} and $R_f = \infty$

Solving for Q1 values r_π, g_m, h_{fe}:

$$h_{fe} = \frac{I_C}{I_B} = \frac{1.26mA}{10.77uA} = \boxed{117} \quad g_m = \frac{I_C}{V_T} = \frac{1.26mA}{25mV} = \boxed{0.05\text{S}} \quad r_\pi = \frac{\beta}{g_m} = \frac{117}{0.05S} = \boxed{2.34\text{k}}$$

Solving for Q2 values r_π, g_m, h_{fe}:

$$h_{fe} = \frac{I_C}{I_B} = \frac{2.19mA}{15.39uA} = \boxed{142} \quad g_m = \frac{I_C}{V_T} = \frac{2.19mA}{25mV} = \boxed{0.088\text{S}} \quad r_\pi = \frac{\beta}{g_m} = \frac{142}{0.088S} = \boxed{1.62\text{k}}$$

For the open-loop response, we continue to keep $R_F = \infty$ and simulate to find the Bode response of the circuit:

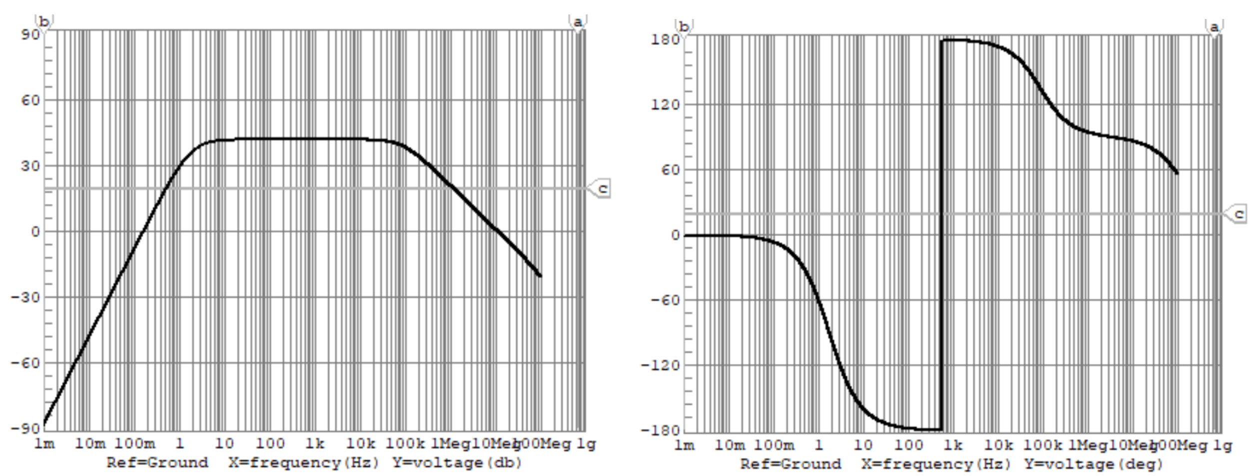

Figure 4.4: Bode Response Magnitude (left) and phase (right)

Upper and lower 3dB frequencies can be found from Figure 4.3 and input/output resistance

@ 1kHz is calculated by adding a test source to the points of resistance testing and using RMS voltage/current.

$$w_{L3dB} = \boxed{\text{2.8Hz}} \quad w_{H3dB} = \boxed{\text{92.6kHz}} \quad A_M = \boxed{\text{-42.13 dB}}$$

$$R_i = \frac{V_{Rms}}{I_{Rms}} = \frac{706uV}{274.3nA} = \boxed{2.58\text{k}\Omega} \quad R_o = \frac{V_{Rms}}{I_{Rms}} = \frac{706uV}{11.3uA} = \boxed{62.26\Omega}$$

We will now transition and set $R_f = 100k\Omega$. However, we will first predict and calculate the values of this closed loop system. Using our parameter type equations for feedback topologies we can use:

$$I_1 = y_{11}V_1 + y_{12}V_2 \quad \rightarrow \quad y_{12} = \frac{I_1}{V_2}|_{V1=0} = -\frac{1}{R_f} = \beta = 10u\Omega$$

$$42.13dB = 127.79V/V \qquad A' = \frac{V_o}{I_s} = R_s\frac{V_o}{V_s} = 5k\Omega * 127.79 = -638.95kV/A$$

Now we have solved for our β and A' we can evaluate our closed loop gain, input/output resistance, and frequency response:

$$A = \frac{A'}{1+A'\beta} = \frac{638.95kV/A}{1+638.95kV/A*10u\Omega} = \boxed{\text{-86.47kV/A}} = \boxed{\text{-17.29V/V}}$$

$$R_i = \frac{R_i'}{1+A'\beta} = \frac{2.58k\Omega}{1+638.95kV/A*10u\Omega} = \boxed{349.14\Omega}$$

$$R_o = \frac{R_o'}{1+A'\beta} = \frac{62.26\Omega}{1+638.95kV/A*10u\Omega} = \boxed{8.43\Omega}$$

$$\omega_{L3dB} = \frac{\omega'_{L3dB}}{1+A'\beta} = \frac{2.8Hz}{1+638.95kV/A*10u\Omega} = \boxed{\text{0.38Hz}}$$

$$\omega_{H3dB} = (\omega'_{H3dB})(1+A'\beta) = (92.6kHz)(1+638.95kV/A*10u\Omega) = \boxed{\text{684kHz}}$$

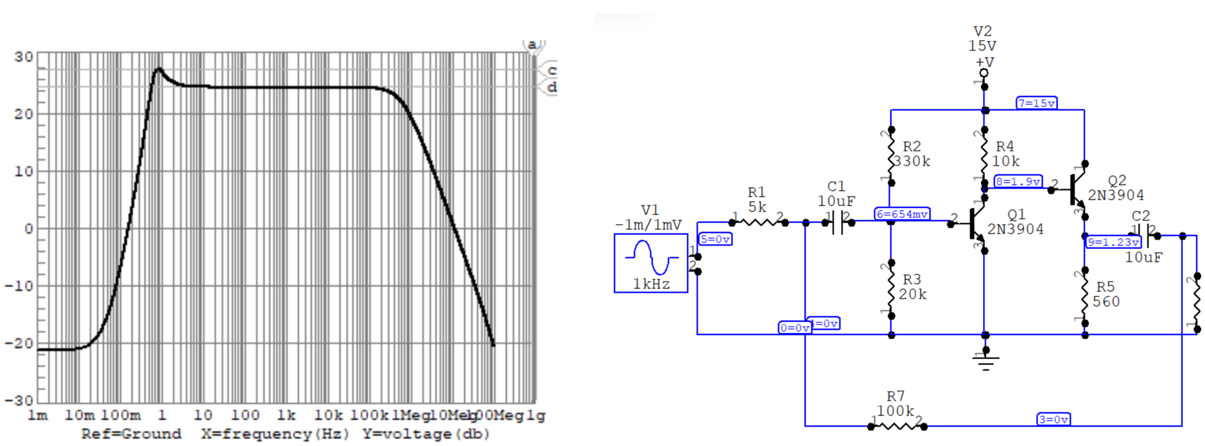

Figure 4.5: Bode Response Magnitude (left) and closed feedback circuit (right)

Methods	A	R_i	R_o	ω_{L3dB}	ω_{H3dB}
Calculated	17.29V/V	349.14Ω	8.43Ω	0.38Hz	684kHz
Measured	17.18V/V	269.12Ω	7.12Ω	510Hz	654kHz

Table 4.3: Calculated vs Measured Closed-Loop System

We can see that our calculations are very accurate and close to our simulated results and that using the parameter type equations for feedback topologies can be really useful. We will now test to see the impact different values of R_f have on the same frequency range (10mHz to 100MHz).

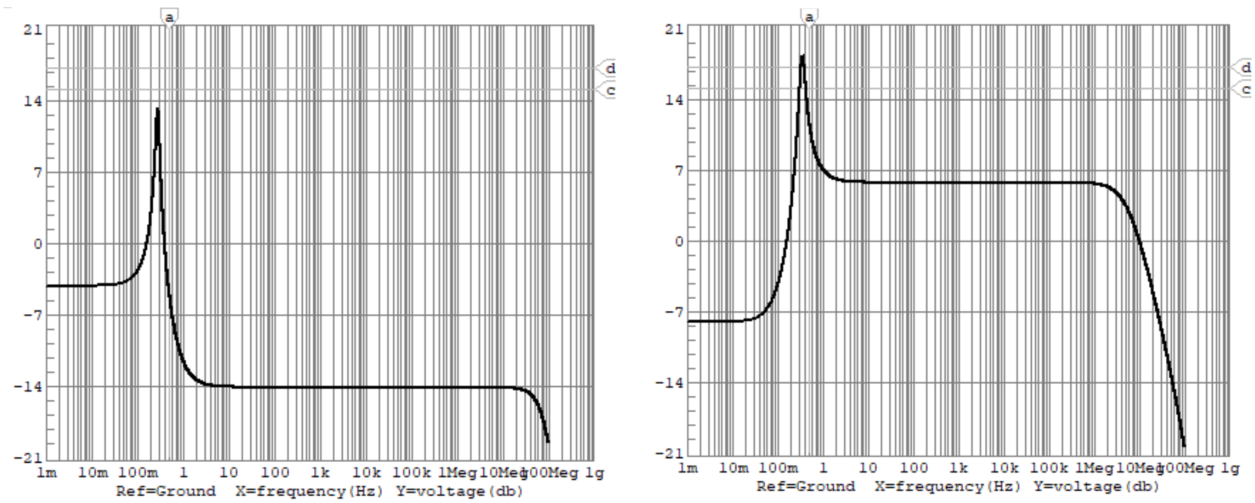

Figure 4.6: Bode Magnitude Response when $R_f = 1k\Omega$ (left) and $R_f = 10k\Omega$ (right)

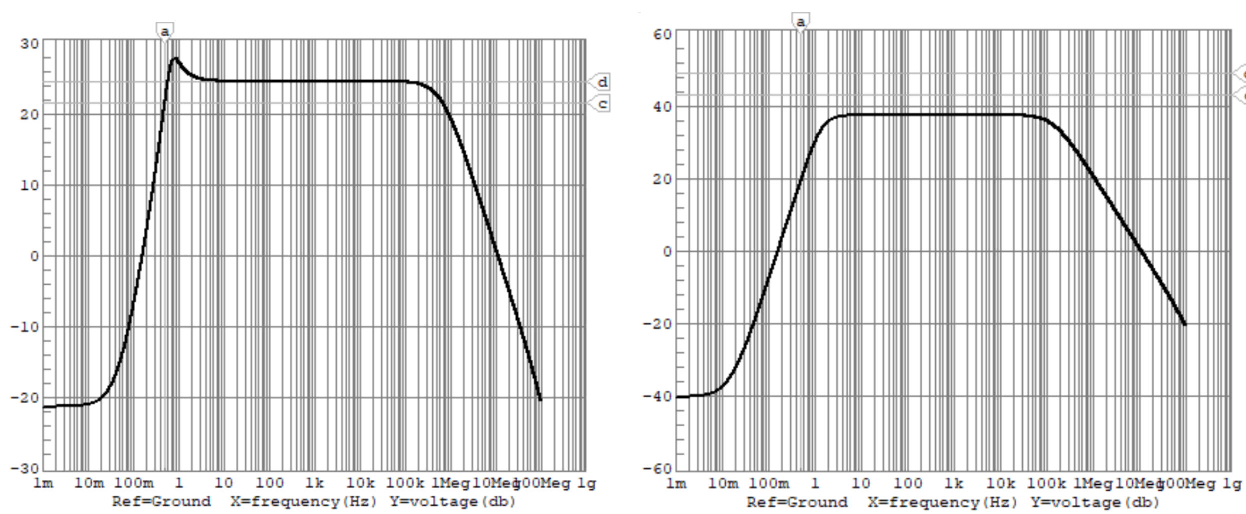

Figure 4.7: Bode Magnitude Response when $R_f = 100k\Omega$ (left) and $R_f = 1M\Omega$ (right)

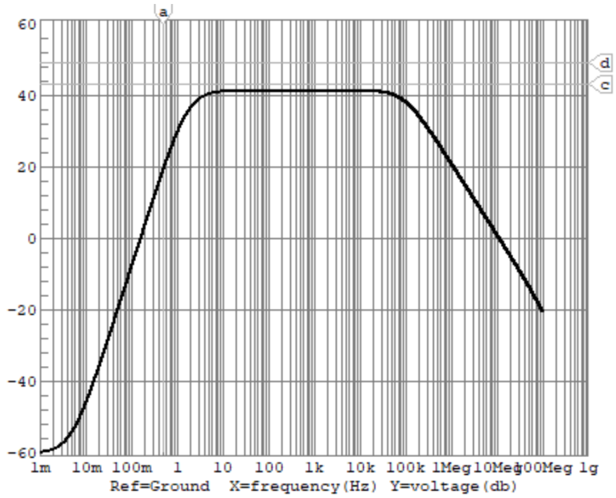

Figure 4.8: Bode Magnitude Response when $R_f = 10M\Omega$ (left)

Now we can compare the impact of the change in R_f. Firstly, the impact on β and A_M from our previous calculations above we know that β is just inverse R_f.

R_f Resistance	A (V/V)	β (Measured)	β (Calculated)
$1k\Omega$	0.2	$1.01E-3$	$1E-3$
$10k\Omega$	1.9	$1.00E-4$	$1E-4$
$100k\Omega$	17.3	$0.99E-5$	$1E-5$
$1M\Omega$	77.6	$1.02E-6$	$1E-6$
$10M\Omega$	120.2	$1.00E-7$	$1E-7$

Table 4.4: Calculated vs Measured β and updated A values

Observing the behavior of the Bode plots, we can see that when R_f is increased the gain goes up. R_f functionally serves as a lever to adjust the gain of this closed loop system. Now how does this impact the input/output resistance? Let's find out!

R_f Resistance	R_o	R_i
$10k\Omega$	1.13Ω	26.48Ω
$100k\Omega$	8.56Ω	241.20Ω
$1M\Omega$	38.06Ω	$1.31k\Omega$

Table 4.6: Esimtated vs Calculated Feedback

This only makes sense. With increased R_f values we get a higher output/input resistance as it is directly influencing the input/output resistance. Now based on these values what is the feedback? We can use the relation: $1 + A\beta = \frac{R_{ii}}{R_i} = \frac{R_{oo}}{R_o}$ as an estimation and using the average of input/output resistance to find the average.

	10kΩ	100kΩ	1MΩ
Estimated Feedback	65.2Ω	7.39Ω	1.64Ω
Simulated Feedback	66.64Ω	9.89Ω	1.78Ω

Table 4.5: Input/Output Resistance with variable R_f + Feedback

Comparing both the estimated and simulated feedback from the circuit we can see that they are both very close so we know our estimation method is quite accurate. We also see that the higher the R_f the lower our feedback is.

The de-sensitivity factor can be calculated by:

$$\text{DS} = |\frac{dA_f/A_f}{dA/A} \quad \text{where} \quad \frac{dA_f}{dA} = \frac{1}{(1+\beta A)^2} \text{ , } A_f = \frac{A}{1+\beta A}$$

We see that when we vary the circuit R_C from $9.9k\Omega$ to $10.1k\Omega$ at $R_f = 100k\Omega$ we get a gain delta (V/V) of 17.26-17.22 = 0.06. Similarly at $R_f = \infty\Omega$ we see a gain delta (V/V) of 128.3 - 126.9 = 1.4. Using the DS formula we can make an estimation of a desensitivity factor of $\boxed{3.19}$.

5 Conclusion

In this lab we have successfully explored oscillators, active amplifiers, and feedback systems.

In part 4: The Feedback Circuit, an interesting thing happened that wasn't obvious, the gain had a peak before 1kHz when the resistance was low. This peak could be due to some added resonance from transistor/source inductance. Capacitance and inductance react at given frequencies to effectively cancel each other out.

References

[1] $https://www.mouser.ca/datasheet/2/389/CD00003223-491114.pdf$

[2] $https://www.youtube.com/watch?v=OrsAtjiChLkab_channel=SalimKoteish$

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