The University of British Columbia

Mini Project 1

Electrical and Computer Engineering

ELEC 301: Electronic Circuits II

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Objectives:

To investigate the accuracy and usefulness of Miller’s theorem and the method of open-circuit and short-circuit time constants and to become familiar with computer based circuit simulation tools.

Part I

A. The Basic Transconductance Amplifier drawn out from MP1 Part I. We will evaluate this circuit to find the locations of its poles, zeros, and find the midband gain.

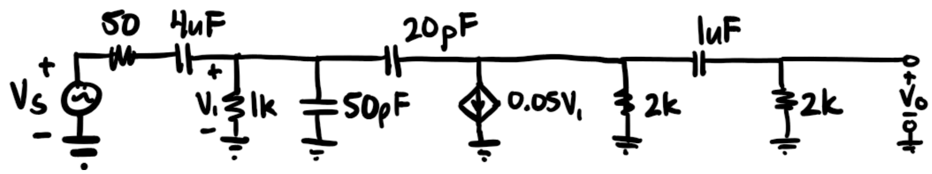


Figure 1.1 Basic Transconductance Amplifier Drawn Out

LOCATING POLES WITH MILLER’S THEOREM

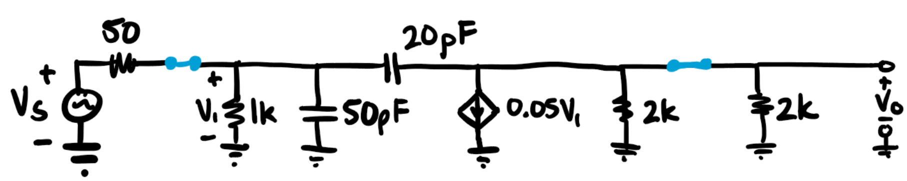
We will first find the poles/zeros of the circuit. Using Miller’s theorem and the method of OC and SC time constants we can analyze Figure 1.1. Below we have Miller’s equations:

$$V_2 = kV_1 \qquad Z_1 = Z\frac{1}{1-k} \qquad Z_2 = Z\frac{k}{k-1} \qquad \text{(Eq.1)}$$

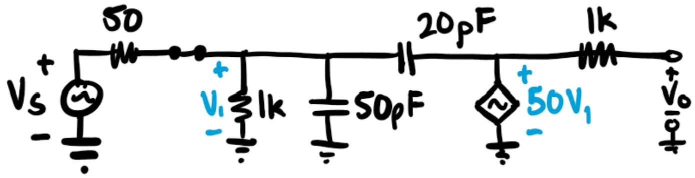
Given we are evaluating just the capacitance, which is the inverse of impedance we can derive the following equations from above:

$$C_{\mu 1} = C_\mu(1-K) \qquad C_{\mu 2} = C_\mu\left(1-\frac{1}{K}\right) \qquad \text{(Eq.2)}$$

Evaluating Figure 1.1 at the midband frequency, we see that our 20pF & 50pF capacitors act as open circuits, while our higher 4uF & 1uF capacitors act as short circuits.



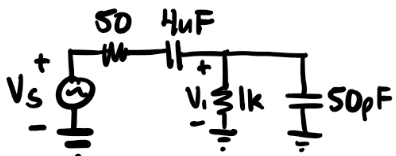
Simplifying and solving for k:

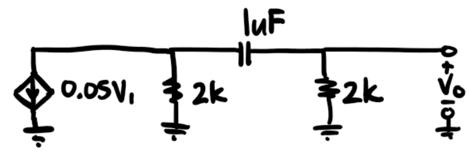


$V_2 = kV_1$

$\boxed{K = -50}$

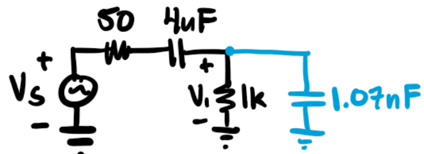
Applying Miller's Formula's (Eq.1) we can split our circuit into two smaller, easier to work with circuits. Applying Miller's Formula (Eq.2) we can find the equivalent capacitance on either side of the two new circuits.

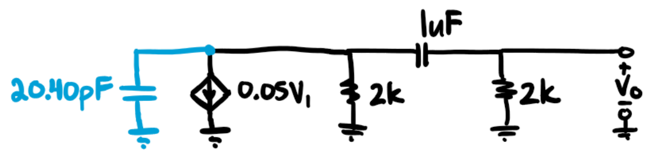




$$C_{\mu 1} = 50pF + 20pF(1-(-50)) = \boxed{1.07nF}$$

$$C_{\mu 2} = 20pF\left(1 - \frac{1}{(-50)}\right) = \boxed{20.40pF}$$





From our simplified circuit we can use the method of OC and SC time constants to find our angular (our poles) and regular frequency:

$$\tau^{oc}_{4uF} = (4uF)(1.05k) = 4.20 \times 10^{-3}$$

$$\omega_{LP1} = \frac{1}{\tau^{oc}_{4uF}} = \frac{1}{4.20 \times 10^{-3}} = \boxed{238 rad/s}$$

$$f_{LP1} = \frac{\omega_{LP1}}{2\pi} = \frac{238 rad/s}{2\pi} = \boxed{37.894 Hz}$$

$$\tau^{oc}_{1uF} = (1uF)(2k+2k) = 4.00 \times 10^{-3}$$

$$\omega_{LP2} = \frac{1}{\tau^{oc}_{1uF}} = \frac{1}{4.00 \times 10^{-3}} = \boxed{250 rad/s}$$

$$f_{LP2} = \frac{\omega_{LP1}}{2\pi} = \frac{250 rad/s}{2\pi} = \boxed{39.789 Hz}$$

$$\tau^{sc}_{1.07nF} = (1.07nF)(1k \| 50) = 5.09 \times 10^{-8}$$

$$\omega_{HP1} = \frac{1}{\tau^{sc}_{1.07nF}} = \frac{1}{5.09 \times 10^{-8}} = \boxed{1.963 \times 10^{7} rad/s}$$

$$f_{HP1} = \frac{\omega_{HP1}}{2\pi} = \frac{1.963 \times 10^{7}}{2\pi} = \boxed{3.124 MHz}$$

$$\tau^{sc}_{2.04pF} = (20.40pF)(2k \| 2k) = 20.4 \times 10^{-9}$$

$$\omega_{HP2} = \frac{1}{\tau^{sc}_{2.04pF}} = \frac{1}{20.4 \times 10^{-9}} = \boxed{4.902 \times 10^{7} rad/s}$$

$$f_{HP2} = \frac{\omega_{HP2}}{2\pi} = \frac{4.902 \times 10^{7}}{2\pi} = \boxed{7.802 MHz}$$

ω_{LP1}	ω_{LP2}	ω_{HP1}	ω_{HP2}
37.894 Hz	39.789 Hz	3.124 MHz	7.802 Mhz

MID BAND GAIN

To find midband gain:

$$A_M = \frac{V_o}{V_s} = \frac{V_o}{V_i} \times \frac{V_i}{V_s}$$

Solving given we know k:

$$V_o = kV_i$$

$$K = \frac{V_o}{V_i} = -50$$

$$= (-50)\left(\frac{1000}{1000+50}\right) = \boxed{-47.619}$$

B. The Basic Transconductance Amplifier from Figure 1.1 imported into CircuitMaker. Point A represents Vo.

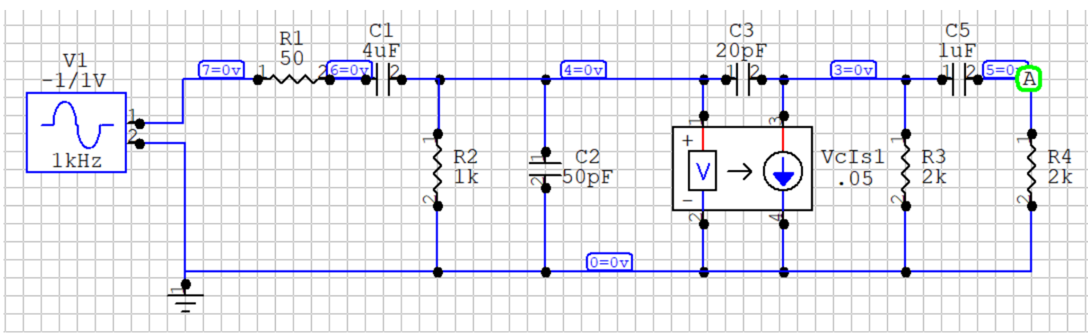


Figure 1.2 Basic Transconductance Amplifier in CircuitMaker

IDENTIFYING POLES VIA SIMULATION

Running an AC simulation of Figure 1.2 from 1mHz to 1THz we can visualize the frequency behavior of the circuit in a Bode plot. Below in Figure 1.3 and Figure 1.4 we have the magnitude and phase response, respectively.

From ELEC 201 we know that each pole/zero contributes a +/- 20dB change in slope on the magnitude of the Bode Plot. We can use straight line segment multiples of 20db/dec to approx pole/zero locations. In this case we have -40dB/dec, -20dB/dec, 0dB/dec, 20dB/dec, and 40dB/dec lines on the Bode magnitude plot to estimate frequency.

For reference +/-20db/dec is illustrated in the top right corner of the graph as a legend.

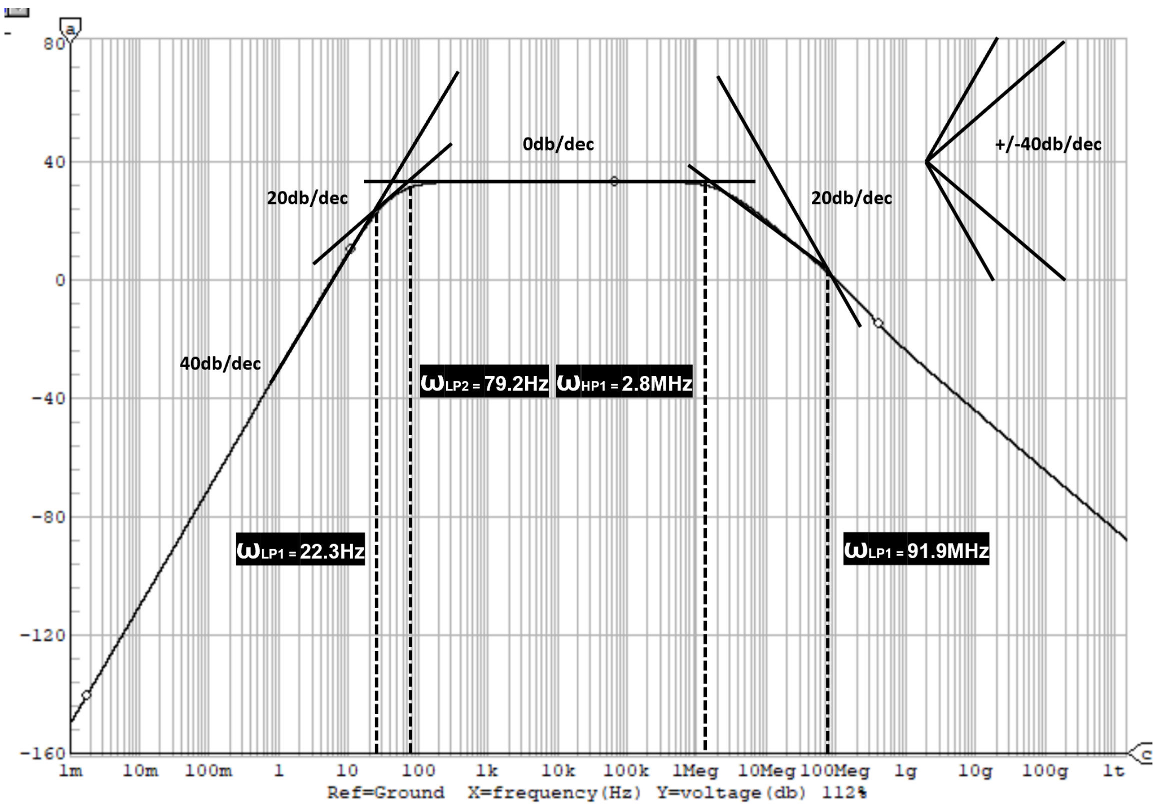


Figure 1.3 Bode Magnitude Plot: Simulation with Approximations

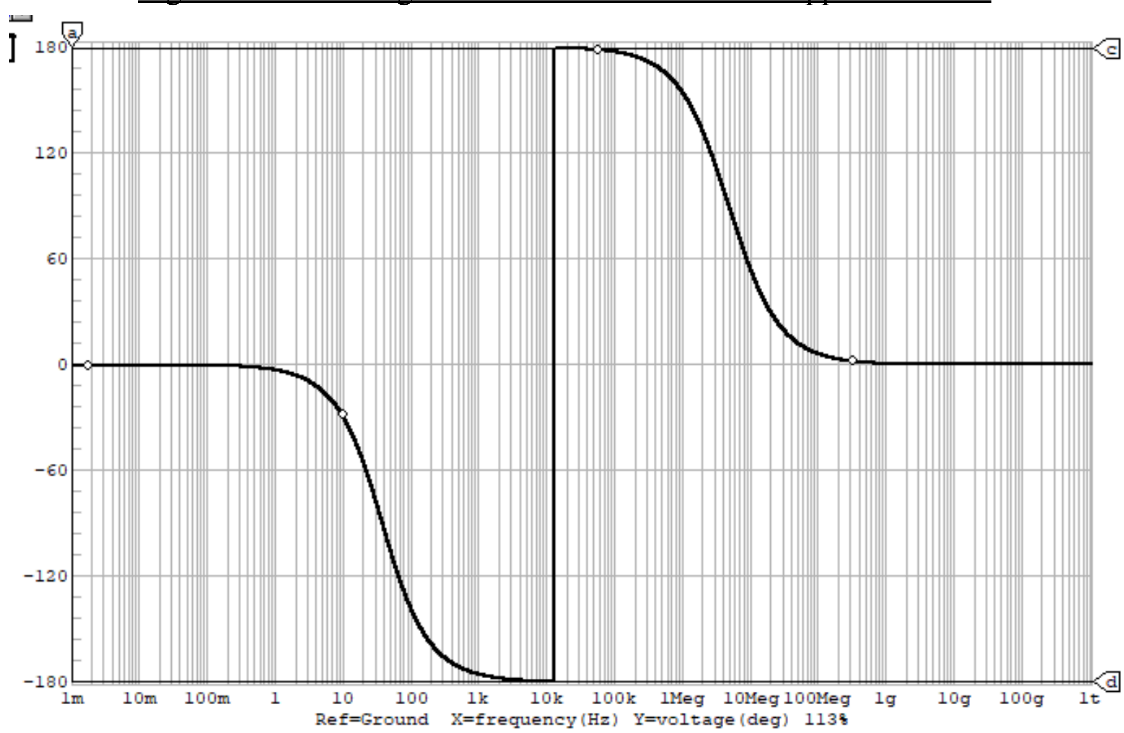


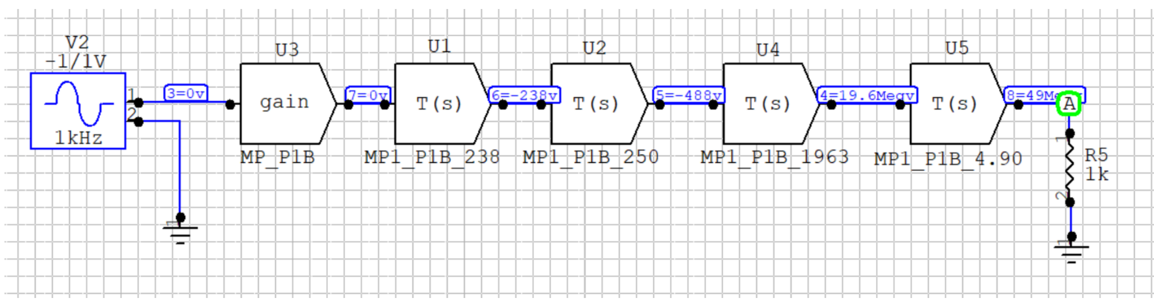
Figure 1.4 Bode Phase Plot: Simulation

SIMULATION VS MILLER'S THEOREM

Using our poles we calculated in Miller's theorem for Part A, we can create an equivalent transfer function for our circuit so we can compare it against our simulation in CircuitMaker.

$$T(s) = -47.619 \times \frac{s}{s + 238\,rad/s} \times \frac{s}{s + 250\,rad/s} \times \frac{1.963 \times 10^{7}\,rad/s}{s + 1.963 \times 10^{7}\,rad/s} \times \frac{4.902 \times 10^{7}\,rad/s}{s + 4.902 \times 10^{7}\,rad/s}$$

Importing into circuit maker:



The result:

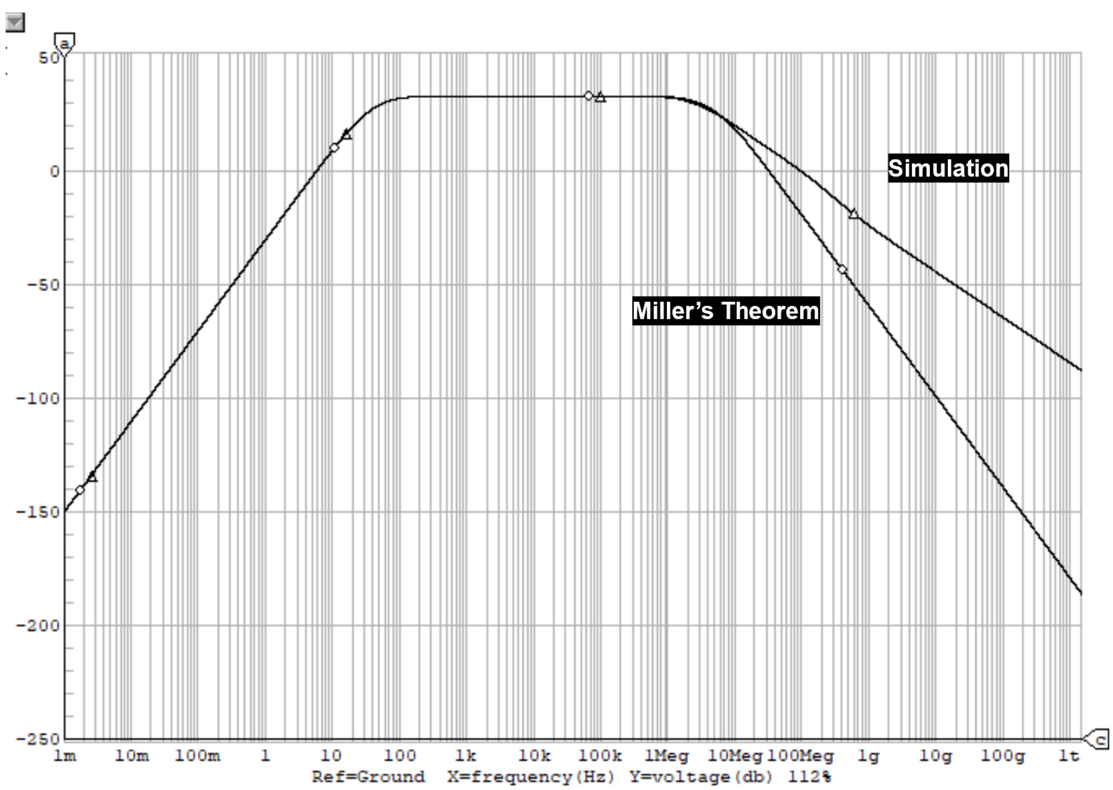


Figure 1.5 Bode Magnitude Plot: Simulation vs Miller's Theorem

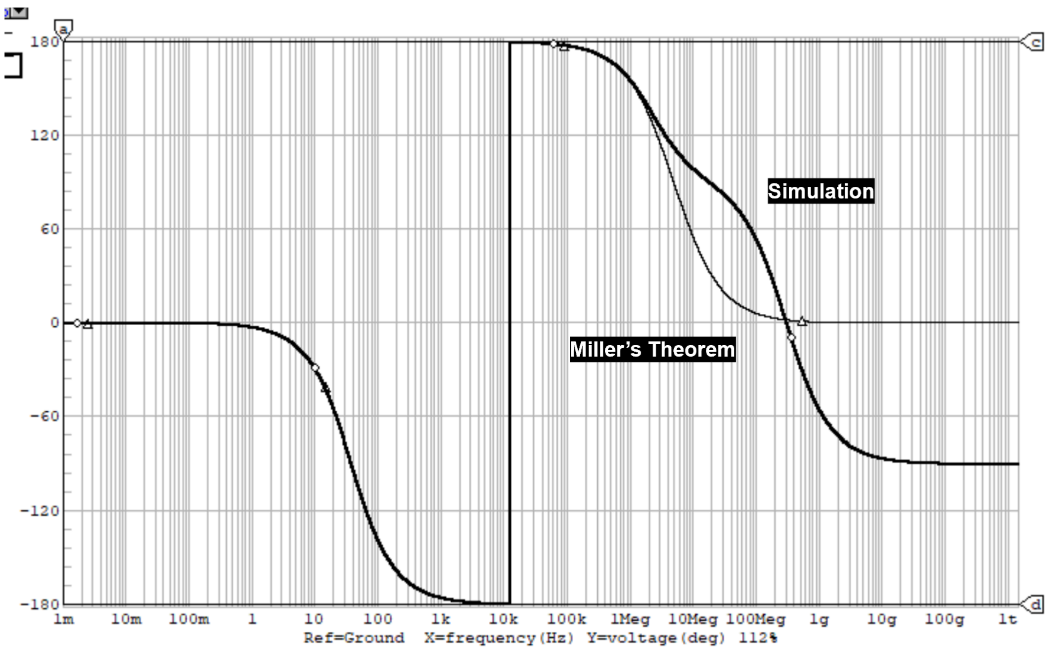


Figure 1.6 Bode Phase Plot: Simulation vs Miller's Theorem

% ERROR: ESTIMATION VS SIMULATION

We use the 3dB points as a reference to calculate the percent error between these methods

Simulation:

Using CircuitMaker and ruler tools to graphically locate. Photo of Graph in Appendix under APPX.1 of an image of this method. Max 15 pages in report.

Millers:

$$\omega_{H3dB} = \frac{1}{\tau_{H3dB}} = \left(\sqrt{\left(\frac{1}{\omega_{HP1}}\right)^2+\left(\frac{1}{\omega_{HP2}}\right)^2}\right)^{-1} = \left(\sqrt{\left(\frac{1}{1.963\times10^7}\right)^2+\left(\frac{1}{4.902\times10^7}\right)^2}\right)^{-1} = 1.822\times10^7\ \text{rad/s}$$

$$= \boxed{2.900\ \text{MHz}}$$

$$\omega_{L3dB} = \sqrt{\omega_{LP1}^2+\omega_{LP2}^2} = \sqrt{238^2+250^2} = 345.172\ \text{rad/s}$$

$$= \boxed{54.936\ \text{Hz}}$$

	Millers 3-dB	Simulated 3-dB	% Error
ω_{L3dB}	54.936Hz	59.400Hz	7.51%
ω_{H3dB}	2.900MHz	2.199MHz	24.17%

Discussion

One hypothesis that can be drawn based on our calculations with Miller's Theorem is that it is very accurate at low frequencies and less accurate at higher frequencies. This is potentially due to the simplification of poles of Miller's Theorem. when compared to simulation based on our 3dB error of 7.51% (low freq error) & 24.17% (high freq error).

Part II

A. A simple four pole RC filter is presented below in Figure 2.1. We will use the method of OC and SC time constants to find the transfer function of such a filter.

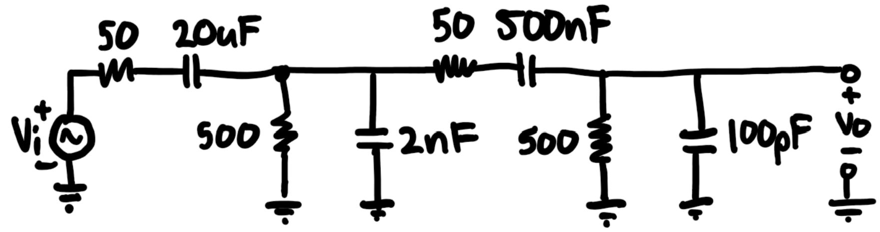


Figure 2.1 Bode Magnitude Plot: Simulation vs Miller's Theorem

IDENTIFYING POLES VIA SIMULATION

First, simulating and extrapolating to find simulated Figure 2.1 poles.

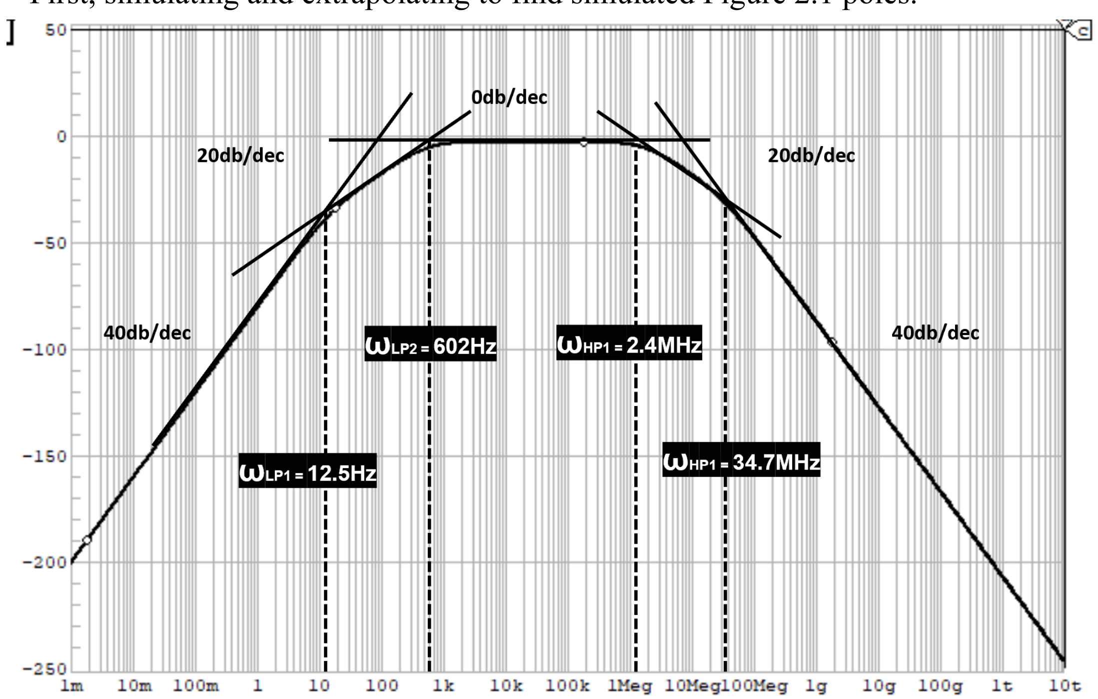


Figure 2.2 Bode Magnitude Plot: Pole Approx - Simulated from Figure 2.1

	ω_{LP1}	ω_{LP2}	ω_{HP1}	ω_{HP2}
100pF	12.5 Hz	602 Hz	2.4 MHz	34.7 Mhz

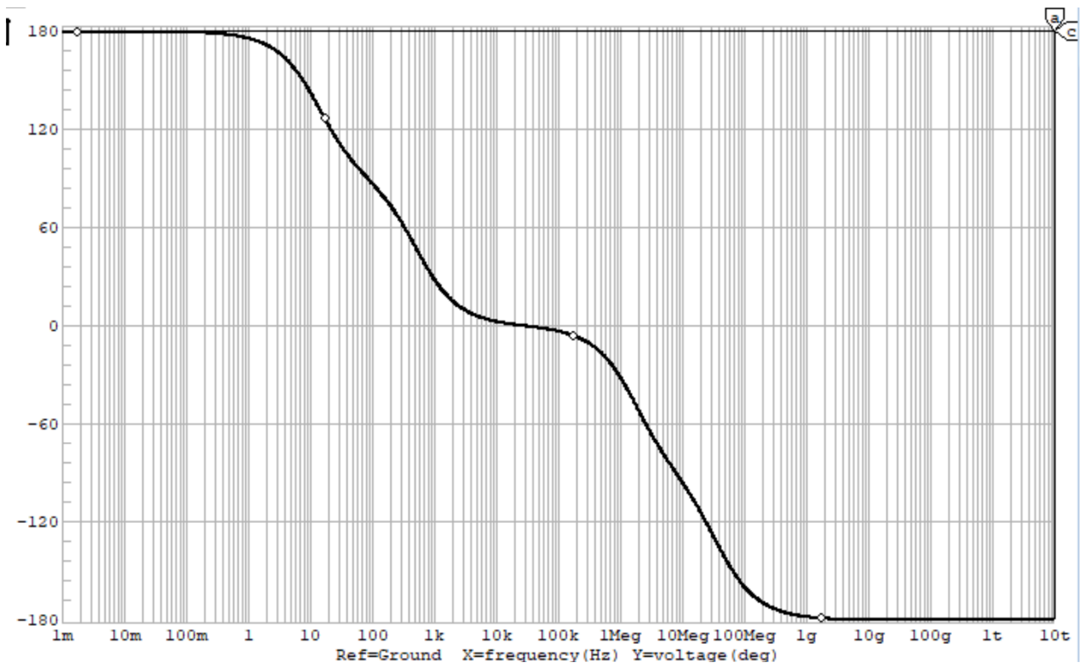


Figure 2.3 Bode Phase Plot - Simulated from Figure 2.1

B. Increasing the value of C_4 to 200pF, 500pF, 1nF, 2nF, 5nF, and 10nF we rerun the AC simulation as well as analyze the circuit using the method of OC and SC time constants and record the effect on the high frequency poles for each C_4 & C_2 value.

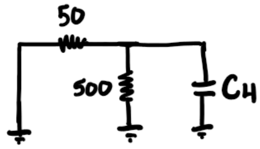
USING OC & SC TO FIND HIGH FREQ. POLES

We can break the circuit from Figure 2.1 down into three circuits given the capacitance. As our range of capacitance for C_4 crosses through C_2, we have to define our OC & SC ranges carefully. *I will use the method described in class to classify OC & SC. A capacitor is "much larger" when it is a factor of 10 larger*

Two circuits for C_4 namely C_2>>C_4 & C_2~C_4 and one for C_2 namely C_2>>C_4.We will evaluate both high frequency caps C_2=2nF, C_4= 200pF-10nF range.

Using the method of OC & SC when at high pole 1, $C_{4.}$

C_2>>C_4: 100pF, 200pF:



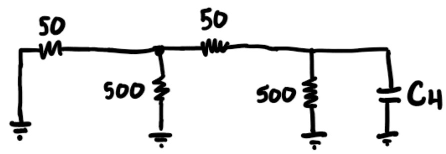
@ $C_4 = 100pF, 200pF$

$R = 50 \| 500 = 45.4545$

$\tau^{sc}_{C4} = (45.4545)(C_4) = 45.4545 C_4$

$\omega_{HP1} = \boxed{\frac{1}{45.4545 C_4}}$ $\quad f_{HP1} = \boxed{\frac{\omega_{HP1}}{2\pi}}$

$C_2 \sim C_4$: 500pF, 1nF, 2nF, 10nF:



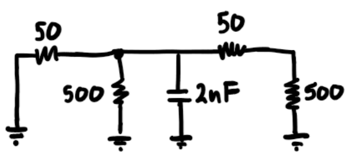
@ $C_4 = 500pF, 1nF, 2nF, 5nF, 10nF$

$R = [(50 \| 500) + 50] \| 500 = 80.153$

$\tau^{sc}_{C4} = (80.153)(C_4) = 80.153 C_4$

$\omega_{HP1} = \boxed{\frac{1}{80.153 C_4}}$ $\quad f_{HP1} = \boxed{\frac{\omega_{HP1}}{2\pi}}$

Using the method of OC & SC when at high pole 2. C_4 is always seen as an OC here.
$C_2 >> C_4$: 100pF, 200pF, 500pF, 1nF, 2nF, 10nF:



@ $C_4 = 100pF, 200pF, 500pF, 1nF, 2nF, 5nF, 10nF$

$R = 550 \| 500 \| 50 = 41.985$

$\tau^{sc}_{C2} = (41.985)(C_2) = 41.985 C_2$

$\omega_{HP2} = \boxed{\frac{1}{41.985 C_2}}$ $\quad f_{HP2} = \boxed{\frac{\omega_{HP1}}{2\pi}}$

Using the equations above:

	ω_{HP1}	ω_{HP2}
100pF	35.014 MHz	1.895 MHz

200pF	17.51 MHz	1.895 MHz
500pF	3.971 MHz	1.895 MHz
1nF	1.986 MHz	1.895 MHz
2nF	992.824 kHz	1.895 MHz
5nF	397.129 kHz	1.895 MHz
10nF	198.565 kHz	1.895 MHz

$$\omega_{H3dB} = \frac{1}{\tau_{H3dB}} = \left(\sqrt{\left(\frac{1}{\omega_{HP1}}\right)^2 + \left(\frac{1}{\omega_{HP2}}\right)^2}\right)^{-1}$$

<u>FINDING 3dB POINTS & ERROR</u>

	ω_{H3dB} (Simulation)	ω_{H3dB} (Calculation)	Error
100pF	1.778 MHz	1.89 MHz	5.93%
200pF	672 kHz	1.88 MHz	64.25%
500pF	572 kHz	1.71 MHz	66.55%
1nF	432 kHz	1.37 MHz	68.48%
2nF	305 kHz	879.47 kHz	65.32%
5nF	132 kHz	388.69 kHz	66.04%
10nF	65.79 kHz	197.48 kHz	66.69%

Discussion

Our results are very interesting. We know that our "ω_{H3dB} (Calculation)" is based on a formula that extrapolates. We see that the estimation is less correct when the poles have frequencies where both are closer to transitioning. For example, at 100pF all other poles are VERY small, so results are accurate, but 500nF is close enough to 100pF and 2nF of the circuit to cause inaccuracies in the calculator.

Part III

A. In Figure 3.1 we have a three-pole low pass filter:

$$T(s) = \frac{V_o(s)}{V_s(s)} = 0.125 \times \frac{10^5/sec}{s + 10^5/sec} \times \frac{10^6/sec}{s + 10^6/sec} \times \frac{10^7/sec}{s + 10^7/sec}$$

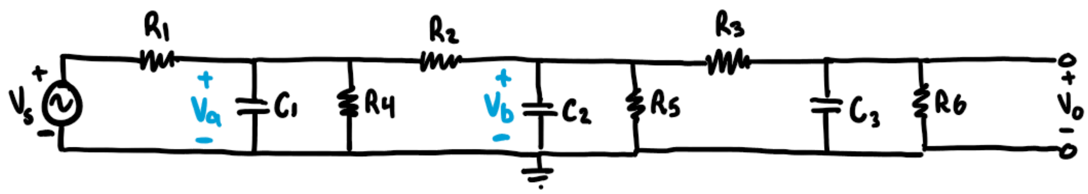


Figure 3.1 Three Pole Low Pass Filter & Transfer Function

Given the following requirements, (1) $C_1>C_2>C_3$ (2) 4x 1k resistors + 2x 2k resistors, we will derive the resistance and capacitance needed in the circuit in Figure 3.1 to satisfy the transfer function in Figure 3.1. We are given the gain from the transfer function, assuming non-unity gain for each node we find:

$$A_m = \frac{V_o}{V_s} = \frac{V_o}{V_b} \times \frac{V_b}{V_a} \times \frac{V_a}{V_s} = \frac{1}{8} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

DERIVING RESISTANCE

Each node has a respective gain of 0.5. With this information we can deduce resistances by solving with the transfer function at each node:

$$\frac{V_o}{V_b} = \frac{R_6}{R_6+R_3} = 0.5 \Rightarrow \boxed{R_6 = R_3}$$

$$\frac{V_b}{V_a} = \frac{(R_3+R_6)||R_5}{(R_3+R_6)||R_5 + R_2} = \frac{2R_6||R_5}{2R_6||R_5 + R_2} = 0.5 \Rightarrow (R_3+R_6)||R_5 = R_2$$

$$\frac{(2k+2k)1k}{4k+1k} \neq 1k, 2k \Rightarrow \boxed{R_6 = R_3 = 1k}$$

$$(R_3+R_6)||R_5 = R_2 \Rightarrow \frac{2kR_5}{2k+R_5}\Big|_{R_5=1k} \neq 1k, 2k \Rightarrow \boxed{R_5 = 2k}$$

$$(R_3+R_6)||R_5 = R_2 \Rightarrow \boxed{R_2 = 1k}$$

$$\frac{V_a}{V_s} = \frac{[(R_3+R_6)\|R_5+R_2]\|R_4}{[(R_3+R_6)\|R_5+R_2]\|R_4+R_1} = \frac{[1k+1k]\|R_4}{[1k+1k]\|R_4+R_1} = \frac{2k\|R_4}{2k\|R_4+R_1} = 0.5 \Rightarrow 2k\|R_4 = R_1$$

$$\left.\frac{2k\cdot R_4}{2k+R_4}\right|_{R_4=1k} \neq 1k, 2k \Rightarrow \boxed{R_4 = 2k} \Rightarrow \boxed{R_1 = 1k}$$

This successfully satisfies requirement (2) -> 4x 1k resistors + 2x 2k resistors.

DERIVING CAPACITANCE

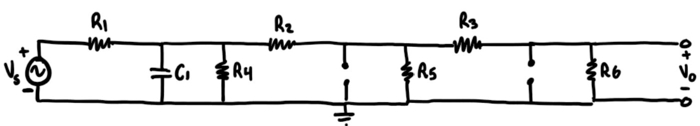
From the transfer function in Figure 3.1 we can also extract tau:

$$T(s) = \frac{V_o(s)}{V_s(s)} = 0.125 \times \frac{10^5/\text{sec}}{s+10^5/\text{sec}} \times \frac{10^6/\text{sec}}{s+10^6/\text{sec}} \times \frac{10^7/\text{sec}}{s+10^7/\text{sec}}$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

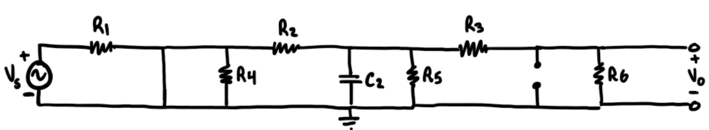
$$\tau_1 = \frac{1}{10^5/\text{sec}} = \boxed{10^{-5}} \quad \tau_2 = \frac{1}{10^6/\text{sec}} = \boxed{10^{-6}} \quad \tau_3 = \frac{1}{10^7/\text{sec}} = \boxed{10^{-7}}$$

Given we know $C_1 > C_2 > C_3$ and $\tau = C \cdot R$ we can assign the following τ values appropriately. Using the method of SC & OC time constants we can find the capacitors from Figure 3.1.



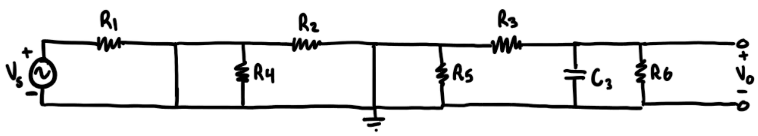
$$\tau_{C_1}^{oc} = [(R_3+R_6)\|R_5+R_2]\|R_4\|R_1 \cdot C_1$$

$$C_1 = \frac{\tau_{C_1}^{oc}}{[(R_3+R_6)\|R_5+R_2]\|R_4\|R_1} = \frac{10^{-5}}{[(1k+1k)\|2k+1k]\|2k\|1k} = \boxed{20nF}$$



$$\tau_{C_2}^{oc} = [(R_3+R_6)\|R_5]\|R_2 \cdot C_2$$

$$C_2 = \frac{\tau_{C_2}^{oc}}{[(R_3+R_6)\|R_5]\|R_2} = \frac{10^{-6}}{[(1k+1k)\|2k]\|1k} = \boxed{2nF}$$



$$\tau_{C3}^{SC} = R_6 || R_3 \cdot C_3$$

$$C_3 = \frac{\tau_{C3}^{SC}}{R_6||R_3} = \frac{10^{-7}}{1k||1k} = \boxed{0.2nF}$$

This successfully satisfies requirement (1) $C_1 > C_2 > C_3$.

BODE PLOT

We now have our final resistance and capacitance values. Our final circuit imported into CircuitMaker seen in Figure 3.2:

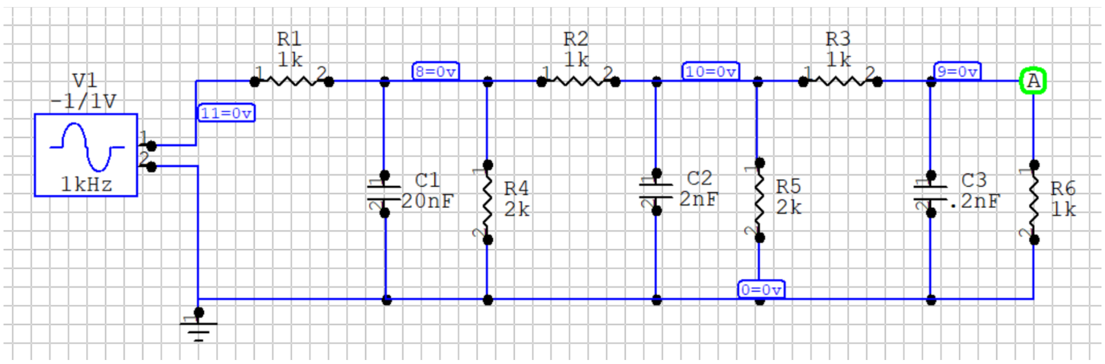


Figure 3.2 Final Circuit in CircuitMaker

Taking the Bode magnitude and phase response:

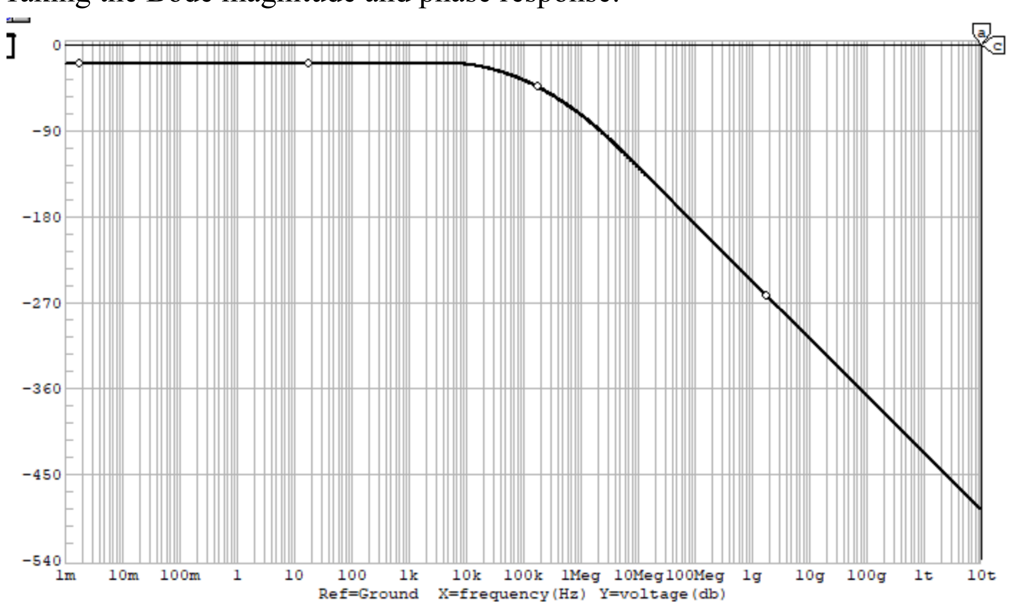


Figure 3.3 Bode Magnitude Plot of Figure 3.2

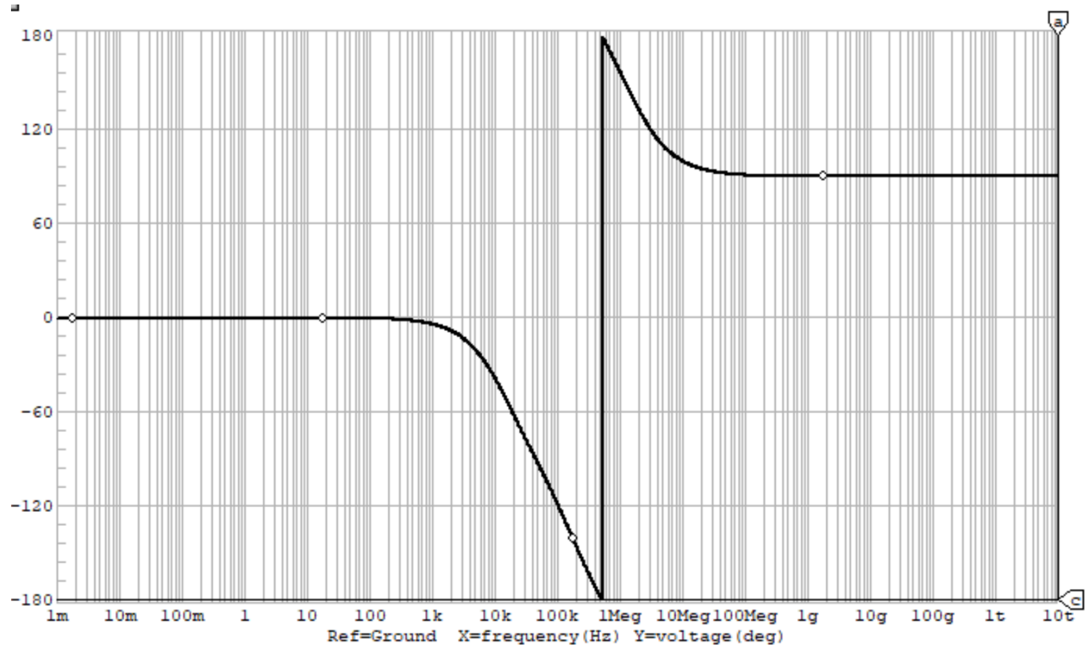


Figure 3.4 Bode Phase Plot of Figure 3.2

B. CIRCUIT VS TRANSFER FUNCTION

Now I will compare this derivation of resistances and capacitances in the Figure 3.2 circuit and simulate it against the transfer function. Taking the transfer function from Figure 3.1:

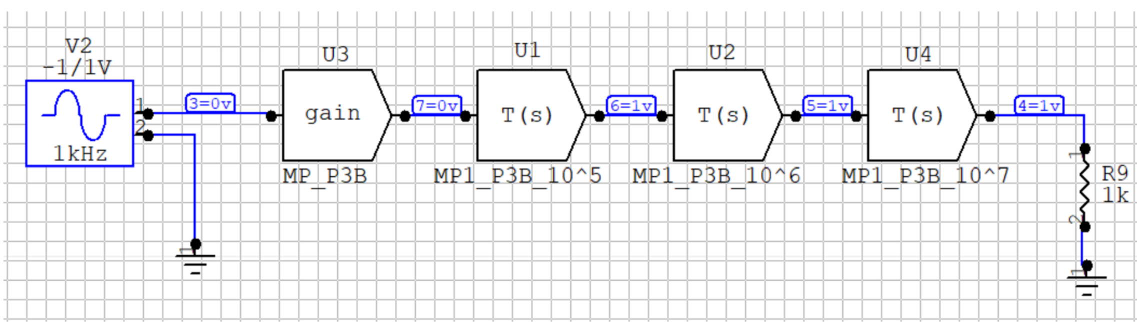


Figure 3.5 Transfer function from Figure 3.1 Simulated in CircuitMaker

Now overlaying for comparison the Bode plots of Figure 3.5 and Figure 3.2:

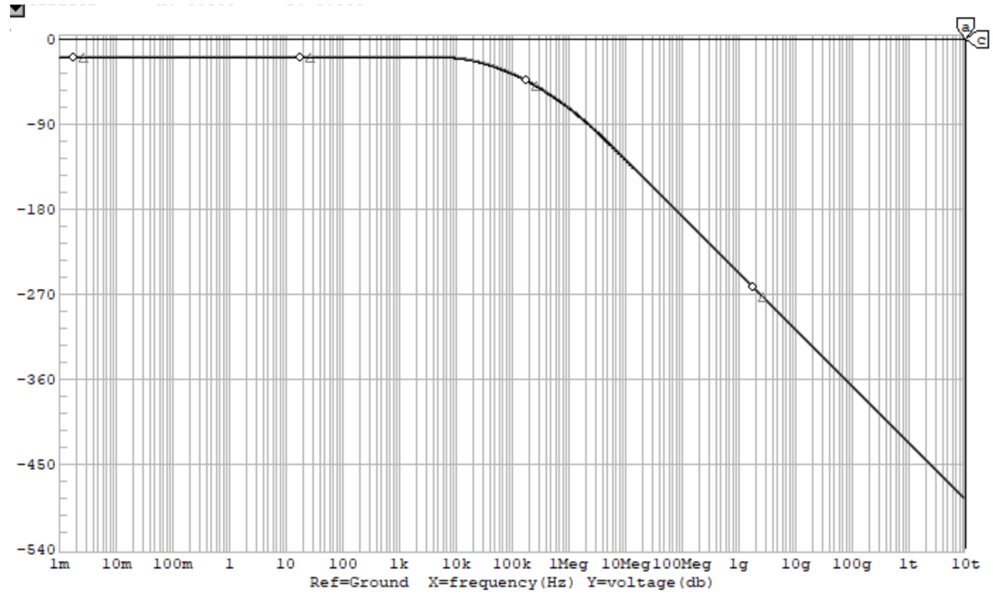


Figure 3.6 Bode Magnitude Plot: Calculated Circuit vs Transfer Function

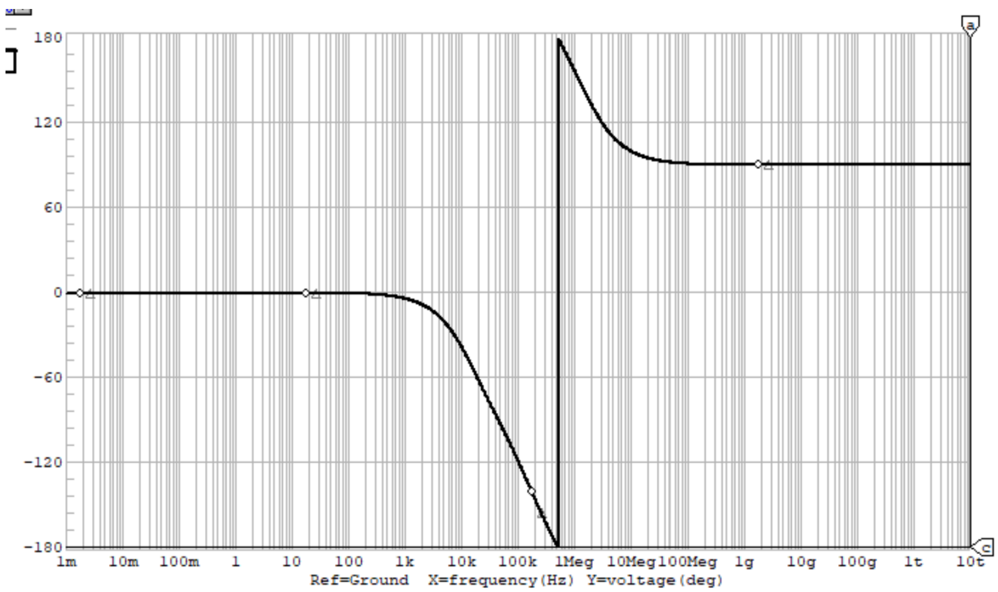


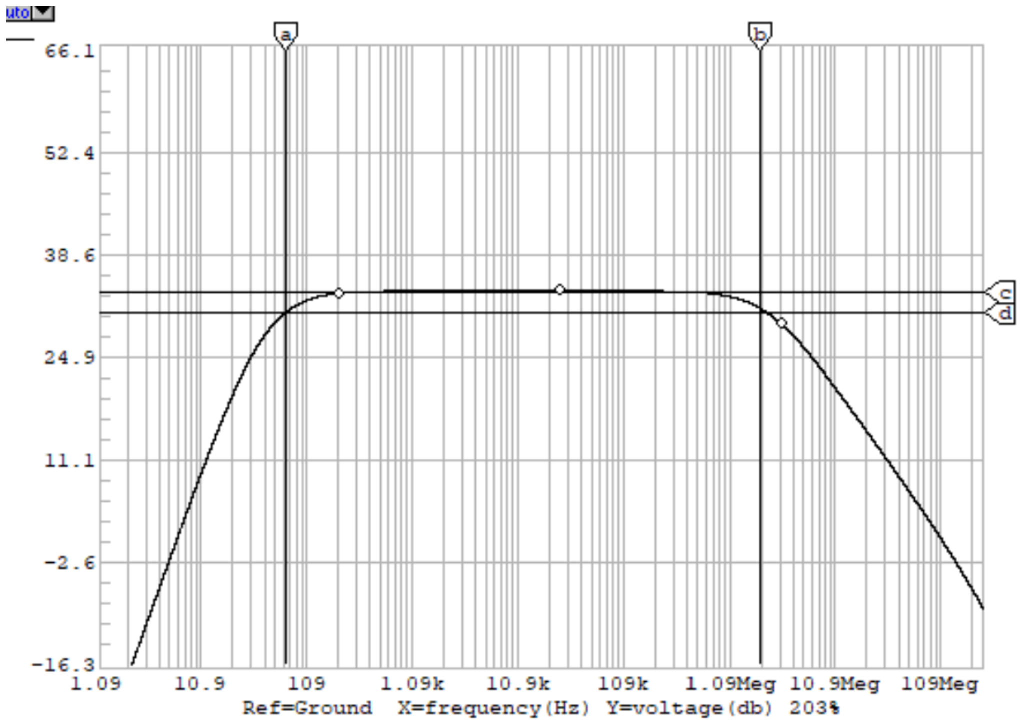
Figure 3.7 Bode Phase Plot: Calculated Circuit vs Transfer Function

Discussion

We can see that the transfer function from Figure 3.1 and our capacitance and resistance that we chose for the circuit Figure 3.2 produce the exact same Bode response. Both the transfer function and circuit **produce the same Bode response.** This effectively demonstrates how you can produce values for a corresponding circuit based solely on a transfer function.

APPENDIX

Part 1B: APPX.1: Finding 3dB from graph



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